PRESSURE DROP IN THE FLOW OF VISCOELASTIC FLUIDS THROUGH FIXED BEDS OF PARTICLES

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The results of experimental investigation of elastic effects in the creeping flow of viscoelastic fluids through fixed beds of spherical and nonspherical particles are presented. A simple form of the corrective Deborah number function has been suggested enabling relations valid for the fixed bed flow of purely viscous fluids to be used for the estimation of pressure drop in the flow of viscoelastic fluids.

The flow of non-Newtonian fluids through fixed beds of particles (granular beds) is a matter of interest in diverse fields of technology and process engineering. The application of this kind of flow is typical e.g. for heterogeneous catalytic chemical and biochemical processes, polymer processing, enhanced oil recovery, and others. In previous paper of Dolejs and Lecjaks[1], a simple model of the flow through fixed beds of particles has been proposed combining the ideas of the flow through capillaries and the flow around solid bodies. The model has been successfully applied to the pressure drop prediction in a creeping flow of purely viscous non-Newtonian fluids through fixed beds of spherical and nonspherical particles[2]. However, during the flow of viscoelastic fluids, the fluid elasticity may manifest itself since the fluid particles are moving through tortuous channels with changing cross-section and the flow is unsteady from Lagrangian point of view. This results in a significant increase of pressure drop in comparison with the pressure drop in the flow of purely viscous fluids at the same flow conditions[3–11]. At present the experimental investigations concerning the resistance to flow of viscoelastic fluids through fixed beds of particles are still in an open stage and there is no general opinion on this problem. Nevertheless, the majority of the investigators used a function of the Deborah number to take into account the influence of fluid elasticity during the flow through the bed. Various forms of the Deborah number and Deborah number corrective function have been proposed[12,13] which provide different prediction of the elastic increase of pressure drop. At the same time, the problem how to express the Deborah number corrective function sufficiently universally for scale-up has not been satisfactory solved yet.

In this contribution, the results are presented of our extensive experimental investigation of the onset and intensity of elastic effects in the creeping flow of viscoelastic fluids through fixed beds of spherical and nonspherical particles. Analysing the pressure drop experimental data, a simple form of the corrective Deborah number function has been suggested enabling relations valid for the fixed bed flow of purely viscous fluids to be used for the estimation of pressure drop in the flow of viscoelastic fluids. The necessary fluid characteristic time has been proposed to be evaluated from fixed bed flow measurements.

THEORETICAL

The Deborah number, which takes into account the influence of the fluid memory on the flow process, has been defined by Reiner[14] as

$$De = \frac{\Theta_f}{\Theta_p} \ , \tag{1}$$

where Θ_f is the fluid characteristic time (relaxation time) and Θ_p is the flow process characteristic time. The elastic pressure drop increase begins to be observable in the creeping flow of viscoelastic liquids through a fixed bed of particles when the Deborah number reaches a limit value De^*. If the characteristic fluid time Θ_f is sufficiently small in comparison with the characteristic time Θ_p of the flow process, so that $De < De^*$, the deviations from the inelastic fluid behaviour are not significant in a fixed bed flow and equations valid for the flow of purely viscous fluids through fixed beds can also be used for the description of the flow of viscoelastic fluids. For $De > De^*$, the pressure drop Δp_{ve} of the viscoelastic fluid in the fixed bed flow can be determined by multiplying the pressure drop Δp_{pv} of the purely viscous fluid (with the same density and viscosity) by a Deborah number corrective function $f(De)$, that is

$$\Delta p_{ve} = \Delta p_{pv} \, f(De) \ . \tag{2}$$

At the same time, the concrete forms of dependence (*2*) proposed in the literature differ according to both the form of relationship used for the calculation of pressure drop Δp_{pv} and the form of Deborah number corrective function $f(De)$. A useful survey of works dealing with this problem was given e.g. by Kemblowski et al.[12].

Our following considerations are based on the modified equation of the Poiseuille flow through a circular pipe

$$D_w = \frac{4}{\tau_w^3} \int_0^{\tau_w} \tau^2 \dot{\gamma}(\tau) \, d\tau \ , \tag{3}$$

which has been proposed by authors[2] for the calculation of a steady laminar flow of purely viscous non-Newtonian fluids through fixed beds of spherical and nonspherical particles. Here

$$\tau_w = \frac{\Delta p_{pv}}{L} l_{ch} \ , \tag{4}$$

$$D_w = \frac{2u_{ch}}{l_{ch}} \tag{5}$$

are the modified dynamical and kinematical variables,

$$l_{ch} = \frac{\varepsilon}{a_p \varphi (1-\varepsilon) + a_w} = \frac{d_p \varepsilon}{6\varphi(1-\varepsilon)M} \tag{6}$$

is the characteristic linear dimension of the bed void space, and

$$u_{ch} = \frac{u}{\varepsilon} \ , \tag{7}$$

$$M = 1 + \frac{2d_p}{3\varphi(1-\varepsilon)D_h} \tag{8}$$

are the fluid characteristic velocity and the corrective coefficient for the wall effect. The quantity φ (bed factor) is a dynamical characteristic of the bed representing a ratio of the total and friction drags of the bed. For purely viscous fluids, the bed factor is independent on rheological properties of the fluids, and has a constant value in the creeping flow region. For a random bed of spherical particles $\varphi = 1.5$, for a bed of nonspherical particles it can be simply determined experimentally using a Newtonian fluid[2]. The pressure drop calculations making use of Eq. (*3*) are the simplest if the power-law model

$$\dot{\gamma} = \left(\frac{\tau}{K} \right)^{\frac{1}{n}} \tag{9}$$

is used for expressing the shear rate function $\dot{\gamma}(\tau)$. In this case, the integration of Eq. (*3*) yields the relationship

$$\tau_w = K\left(\frac{3n+1}{4n} D_w\right)^n \quad . \tag{10}$$

The relationship (*10*) can also be expressed in terms of the bed drag coefficient f_{pv} as

$$f_{pv} \equiv \frac{\Delta p_{pv}\, d_p \varepsilon^3}{\rho_f u^2 L(1-\varepsilon)} = \frac{72\,(\varphi M)^{n+1}}{Re_n} \quad , \tag{11}$$

where

$$Re_n = \frac{\rho_f\, u^{2-n} d_p^n}{K} \frac{\varepsilon^{2(n-1)}\, 12^{1-n}}{(1-\varepsilon)^n} \left(\frac{4n}{3n+1}\right)^n \tag{12}$$

is the modified Reynolds number for power-law fluid flowing through fixed bed. Considering the relationships (*2*), (*4*), (*10*), and (*11*), the dynamical variable $\tau_{w,ve}$ for the flow of viscoelastic fluids through fixed beds can be expressed as

$$\tau_{w,ve} \equiv \frac{\Delta p_{ve}}{L} l_{ch} = K\left(\frac{3n+1}{4n} D_w\right)^n f(De) \tag{13}$$

or the corresponding bed drag coefficient f_{ve} as

$$f_{ve} \equiv \frac{\Delta p_{ve}\, d_p\, \varepsilon^3}{\rho_f\, u^2 L\, (1-\varepsilon)} = \frac{72\,(\varphi M)^{n+1}}{Re_n} f(De) \quad . \tag{14}$$

In a number of published correlation equations[4–9], the Deborah number function

$$f(De) = 1 + A\, De^B \tag{15}$$

theoretically derived by Wissler[4] with $B = 2$, has been used. However, the choice of suitable form of Deborah number and values of coefficient A and Deborah number exponent B remains still an open problem.

The form of the Deborah number depends on both the expression of the flow process characteristic time and the expression of the fluid characteristic time.

The flow process characteristic time Θ_p is taken to be a time interval during which a fluid element experiences a significant sequence of kinematics events. Metzner et al.[15] showed that for a fixed bed flow the time Θ_p is directly proportional to a characteristic path L_{ch} in the flow direction and indirectly proportional to a characteristic fluid

velocity U_{ch}. Often the velocity u_{ch} is substituted for U_{ch} and the effective particle diameter d_p for L_{ch} do that

$$\Theta_{p,1} = \frac{d_p}{u_{ch}} \quad . \tag{16}$$

According to our approach to the fixed bed flow problem, the characteristic linear dimension l_{ch} is used for L_{ch}, therefore

$$\Theta_{p,2} = \frac{l_{ch}}{u_{ch}} = \frac{2}{D_w} \quad . \tag{17}$$

The determination of the characteristic fluid time Θ_f is not easy at all. In the papers dealing with the fixed bed flows, the use of the characteristic fluid time given as

$$\Theta_{f,1} = \frac{\Psi_1(\dot{\gamma})}{2\eta(\dot{\gamma})} \tag{18}$$

prevails[4–9]. In expression (*18*), which follows from the White–Metzner viscoelasticity model[16], $\Theta_{f,1}$ represents the material function depending on shear rate $\dot{\gamma}$. At $\dot{\gamma} \to 0$ both the first normal stress coefficient Ψ_1 and the viscosity η are constant. Therefore, the limit

$$\Theta_{f,0} = \lim_{\dot{\gamma}\to 0}\left(\frac{\Psi_1(\dot{\gamma})}{2\eta(\dot{\gamma})}\right) = \frac{\Psi_{1,0}}{2\eta_0} \tag{19}$$

is also constant, and in this way we can obtain a constant value of the characteristic fluid time. According to Tanner[17] and Astarita[18], the limit (*19*) is determining because under low shear rate flow conditions, the White–Metzner viscoelasticity model is valid for the most of viscoelastic fluids.

A variety of other possible expressions of Θ_f based on more complicated viscoelasticity models are discussed in literature[16]. For example, the six parameter Spriggs model has been used by Smirnov[19] for the estimation of the characteristic fluid time. However, there is a problem in measuring other material functions occurring in the more complicated viscoelasticity models.

Gaitonde and Middleman[20] and Kemblowski et al.[21] used for the correlation of their fixed bed flow results the characteristic fluid time based on Bueche's molecular theory, the Rouse chain model[22] has also been applied (e.g.[10,23]).

The simplest possibility how to choose (at least formally) the characteristic time of the fluid is founded on the use of a time parameter following from a fluid viscosity

model[16]. For the flow of polymeric liquids through fixed beds, the characteristic time $\Theta_{f,2} = \eta_0/\tau_{1/2}$ based on the parameters of the Ellis model has been used[11,24]. According to authors[16,25], the parameter λ occurring in the Carreau A model is also closely related to the characteristic time constant for most polymeric fluids.

The value of the coefficient *A* in Eq. (*15*) naturally depends on the form of the Deborah number chosen. However, the results of the published works show that even for the Deborah numbers defined by the same way, the coefficient *A* has not the same value for each viscoelastic fluid. For example, for

$$De_{1,1} = \frac{\Theta_{f,1}}{\Theta_{p,1}} = \frac{\Psi_1(\dot{\gamma})}{2\eta(\dot{\gamma})} \frac{u}{\varepsilon d_p} \qquad (20)$$

the coefficient *A* takes values from 8 to 90. In connection with the variable value of the coefficient *A*, the limit value $De^*_{1,1}$ of Deborah number $De_{1,1}$, which corresponds to the onset of elastic increase in the bed resistance, will also acquire different values[4,5,7] (e.g. 0.03–0.1).

Therefore, the reliability of the various forms of the Deborah number and Deborah number corrective function expressions proposed for the prediction of pressure drop in the flow of viscoelastic fluids through fixed beds of particles should be verified by comparing with a set of experimental data as extensive as possible.

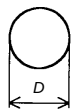
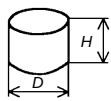
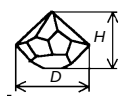
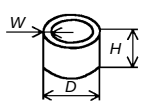
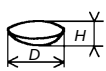
EXPERIMENTAL

In our fixed bed flow experiments, the dependence of the pressure drop on the volume flow rate in the flow of viscoelastic polymer solutions through fixed beds of spherical and non-spherical particles was measured. In total, 47 different fixed bed–polymer solution systems were tested.

Glass spheres, cylinders, polyhedrons, cubes, Rasching rings, lenses, and sand particles were used for preparing the beds. The characteristics of particles used are given in Table I. The non-Newtonian fluids were aqueous solutions of hydroxyethyl cellulose (Natrosol 250 MR, Hercules Powder Co., The Netherlands), polyethylene oxide (Polyox WSR 301, Union Carbide, U.S.A.), polyacrylamide (Separan AP-45, Dow Chemical Co., U.S.A., and Kerafloc A4008, Zschimmer–Schwarz Chemischen Fabriken, Germany), and dilute solutions of Separan AP-45 in starch sugar syrup (Boger fluids). The shear stress–shear rate data were measured using the cylindrical rotary rheometer Rheotest II and using a special piston capillary rheometer[26] at low shear rates. The normal stress differences were measured using the rheogoniometer R-18 (at TH Leuna-Merseburg laboratory). Typical courses of the shear stress–shear rate and primary normal stress difference–shear rate dependencies are shown in Figs 1 and 2. The power law parameters *n*, *K*, and *m*, K_N of the test fluids (Table II) were evaluated from the viscometric data in the range of shear rates corresponding to those reached in the fixed bed experiments. This range was estimated according to the relationship

$$\dot{\gamma} = \frac{3n+1}{4n} D_w \qquad (21)$$

TABLE I
Characteristic of particles used

Particle No.	Shape	Dimensions mm	ρ_p kg m^{-3}	a_p m^{-1}	φ
1	Spheres	D = 1.00	2 700	6 000	1.53
2		D = 1.46	2 500	4 110	1.46
3		D = 1.78	2 515	3 371	1.50
4		D = 1.92	2 527	3 120	1.46
5		D = 2.96	2 864	2 439	1.50
6		D = 2.67	2 836	2 247	1.54
7		D = 2.78	2 504	2 158	1.46
8		D = 3.22	2 846	1 863	1.52
9		D = 4.09	2 521	1 467	1.52
10	Cylinders 	D = 2.70 H = 3.30	902	2 080	1.42
11	Polyhedrons 	D = 4.00 H = 2.70	2 863	1 938	1.78
12	Cubes 	H = 5.60	2 227	1 080	1.28
13	Rashig rings 	D = 5.80 H = 5.20 W = 1.30	2 328	1 870	1.67
14	Lenses 	D = 5.1 H = 2.1	2 250	1 270	1.50
		Range of sieve mesh		$a_p\varphi$	
15	Sand particles 	0.50–0.71	2 644	19 319	
16		0.71–1.00	2 630	15 045	
17		1.25–1.60	2 611	8 834	
18		1.60–2.00	2 606	7 120	
19		2.80–4.00	2 570	3 951	

substituting into (*21*) the corresponding maximum and minimum experimental values for the kinematical variable D_w.

The pressure drop experiments were carried out in two horizontal cylindrical columns of 2 cm and 4 cm in diameter and 30 cm and 50 cm in length, respectively. A pressured reservoir (montejus) was used to force the polymer solutions through the beds of particles. The pressure drop was measured making use of two tensometric transducers. The distance between the pressure taps was 20 cm and 40 cm, respectively. During experiments the flow rates were measured by weighting the liquid collected in a known time interval at a constant pressure drop level. The pressure drop was adjusted by the pressure regulation valve situated between the pressure gas cylinder and the montejus from its lowest value to the highest one, and then for several (2–3) measurements, in reversal mode to check the reproducibility of the experimental data. On the whole, 10 experimental ($\Delta p_{ve} - D_w$) data couples were obtained for each system tested. The experiments are described in more detail in ref.[13].

RESULTS AND DISCUSSION

Based on the experimental data of pressure drop Δp_{ve} and superficial velocity u, the variables D_w and $\tau_{w,ve}$ were calculated making use of Eq. (*5*), and left-hand side term of Eq. (*13*), respectively, for each system investigated. Simultaneously, the corresponding theoretical values of purely viscous variable τ_w were determined according to Eq. (*10*). The necessary values of the bed factor φ are given in Table I for individual fixed beds of particles tested[2]. The accuracy of these values was also verified by the pressure drop measurements during the flow of glycerol through the beds. Then, the plots of experimental dependencies of $\tau_{w,ve}$ upon D_w along with the nominal flow curves representing the course of relationship (*10*) were drawn for each test polymer solution. Typical examples of the dependencies obtained are shown in Fig. 3 for the flow of 0.35 wt.%

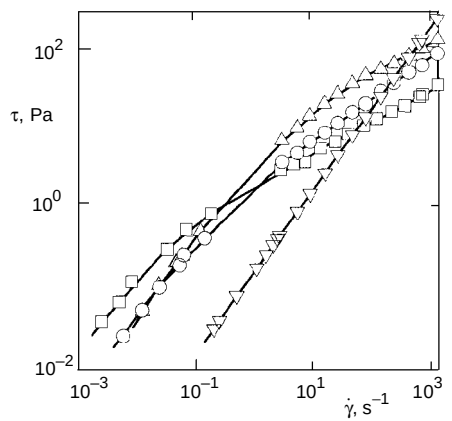


FIG. 1
Shear stress τ as a function of shear rate $\dot{\gamma}$ for test polymer solutions: ❍ 1.4% Polyox, ❒ 0.5% Separan, Δ 1.4% Natrosol, ∇ Boger fluid I

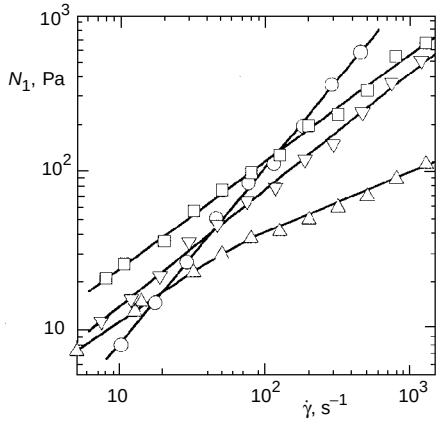


FIG. 2
First normal stress difference N_1 as a function of shear rate $\dot{\gamma}$ for test polymer solutions: ❍ 1.4% Polyox; ❒ 0.5% Separan; Δ 1.4% Natrosol; ∇ Boger fluid I

solution of Separan AP-45 and 1.4 wt.% solution of Polyox WSR 301 through the beds of spherical and non-spherical particles.

In accordance with expectation, the results obtained confirm good agreement between theoretical purely viscous and experimental pressure drop data at the low values of D_w. In this region, the elastic effects can be neglected and Eqs (*10*) or (*11*) are valid. A limit value of variable D_w^* can be observed over which the elastic increase of pressure drop begins to be evident and the deviations of Δp_{ve} from their corresponding purely viscous values Δp_{pv} are increasing. This reality can also be seen in Fig. 4 in which the ratio $\Delta p_{ve}/\Delta p_{pv}$ is drawn in dependence on variable D_w for some systems investigated. In our experiments, the ratio $\Delta p_{ve}/\Delta p_{pv}$ rose as much as six times. At the same time the value of D_w^* is in accordance with the Eqs (*1*) and (*17*) directly proportional to De^*.

It is also evident from Figs 3 and 4 that the elastic pressure drop increase is not affected by the shape of particles forming the fixed beds. This supports the suitability of Eq. (*17*) proposed by us for the estimation of the flow characteristic time $\Theta_{p,2}$ for both spherical and non-spherical particles. The different results mentioned in ref.[9] have been apparently caused by degradation of elasticity of the test polymer solution during measurements in various beds of particles. In our set of fixed bed flow experiments, the incidental degradation of polymer solutions elasticity was checked by the pressure drop measurements in the standard bed of spherical particles both at the beginning and at the end of each series of experiments.

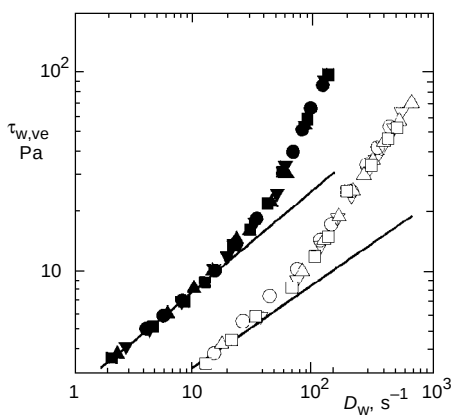


FIG. 3

Experimental dependence of variable $\tau_{w,ve}$ upon variable D_w for 0.35% Separan (empty symbols) and 1.4% Polyox (full symbols): Fixed bed: ○, ● spheres (d = 1.46 mm); Δ, ▲ spheres (d = 2.78 mm); ❐, ■ cylinders; ∇, ▼ polyhedrons. Full lines refer to the nominal flow curves (Eq. *10*)

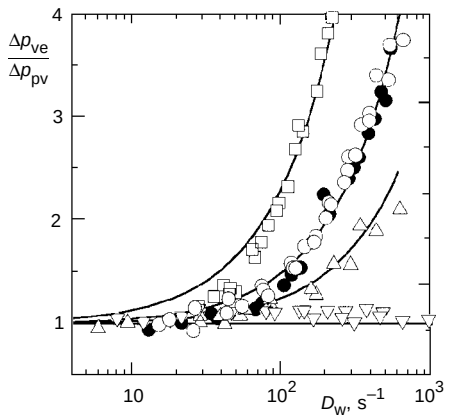


FIG. 4

Dependence of ratio ($\Delta p_{ve}/\Delta p_{pv}$) on variable D_w. ○, ● 0.35% Separan; ❐ Boger fluid I.; Δ 0.4% Kerafloc; ∇ 1.4% Natrosol. Empty symbols refer to beds of spherical particles, full symbols to beds of non-spherical particles

For the verification of the form (*15*) of Deborah number corrective function, the experimental dependencies log $(\tau_{w,ve}/\tau_w - 1)$ versus log *De* were plotted. If the relationship (*15*) with a constant coefficient *A* is generally valid, the above-mentioned dependencies should lie on a single straight line of slope $B = 2$ for all systems investigated. An example of these dependencies evaluated from our experimental pressure drop data using the Deborah number expressed as

$$De_{1,2} = \frac{\Theta_{f,1}}{\Theta_{p,2}} = \frac{\Psi_1(\dot{\gamma})}{4\eta(\dot{\gamma})} D_w \qquad (22)$$

is shown in Fig. 5. The scatter of experimental data indicates that the coefficient *A* and Deborah number exponent *B* in relationship (*15*) are not constant and the kind of polymeric fluid has influence on their values. In the case of Borger fluids, the Deborah number exponent is even negative. Therefore, the use of the characteristic fluid time $\Theta_{f,1}$ for the expression of elastic behaviour of viscoelastic fluids in the flow through fixed beds is not generally adequate, particularly for Boger fluids.

Better results of correlation of pressure drop experimental data were obtained when the zero shear rate limit $\Theta_{f,0}$ of the characteristic fluid time was used for the evaluation of the Deborah number. The values of $\Psi_{1,0}$ and η_0 needed to this purpose were estimated from the flow curves course[13]. The examples of dependencies log $(\tau_{w,ve}/\tau_w - 1)$

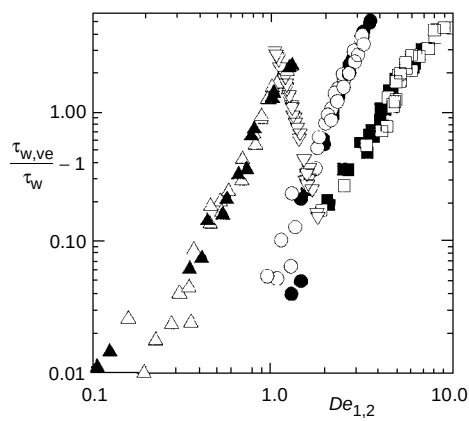


FIG. 5

Dependence of $(\tau_{w,ve}/\tau_w - 1)$ on Deborah number $De_{1,2}$ (Eq. *22*). ○, ● 0.35% Separan; ❐, ■ 0.5% Separan; ∇ Boger fluid I; Δ, ▲ 1.4% Polyox. Empty symbols refer to beds of spherical particles, full symbols to beds of non-spherical particles

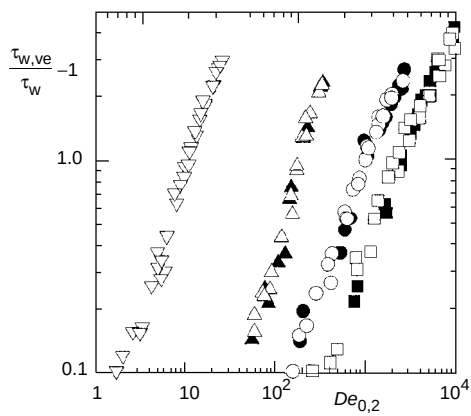


FIG. 6

Dependence of $(\tau_{w,ve}/\tau_w - 1)$ on Deborah number $De_{0,2}$ (Eq. *23*). ○, ● 0.35% Separan; ❐, ■ 0.5% Separan; ∇ Boger fluid I; Δ, ▲ 1.4% Polyox. Empty symbols refer to beds of spherical particles, full symbols to beds of non-spherical particles

versus log $De_{0,2}$ obtained for fixed bed flow of solutions of Separan AP-45, Polyox WSR 301, and for the flow of Boger fluid are shown in Fig. 6. Here

$$De_{0,2} = \frac{\Theta_{f,0}}{\Theta_{p,2}} = \frac{\Psi_{1,0}}{4\eta_0} D_w \quad . \tag{23}$$

In this case, the identical value of Deborah number exponent $B = 1$ can be taken for all systems measured. A shift in location of experimental data for individual test fluids, however, gives evidence that the characteristic fluid time alone is not sufficient for a general description of the flow situation investigated. To this purpose, the value of coefficient A must also be available for a viscoelastic fluid.

The same conclusions result if other before mentioned fluid characteristic time constants are utilised instead of the time $\Theta_{f,0}$. For example, Vorwerk and Brunn[23] have drawn the conclusion that the Deborah number concept based on the characteristic fluid constant following from the Rouse chain model cannot be applied to a porous medium flow of polymer solutions of higher concentration.

Considering the fact that the fluid elastic response depends on the mode of deformation[27], the natural possibility arises to determine an effective characteristic fluid time using the fixed bed of particles directly as a porous rheometer[10,17]. The Deborah number can be in this case expressed as

$$De_{e,2} = \frac{\Theta_{f,e}}{\Theta_{p,2}} = \frac{\Theta_{f,e} D_w}{2} \quad . \tag{24}$$

The effective characteristic fluid time $\Theta_{f,e}$ can be developed from the value of the variable D_w at the onset of elastic increase of bed resistance. Since the value of Deborah number exponent $B = 1$ was found, the corrective function of Deborah number $De_{e,2}$ can be written as

$$f(De_{e,2}) = 1 + A\, De_{e,2} = 1 + A \frac{\Theta_{f,e} D_w}{2} \quad . \tag{25}$$

As the same time, the value of the product $A\, \Theta_{f,e}$, which is supposed to be characteristic for the fluid elastic behaviour in the flow through fixed beds, is determined by the value of D_w^* at the onset of elastic pressure drop increase. Let us suppose that the elastic pressure drop Δp_{ve} exceeds the corresponding purely viscous pressure drop Δp_{pv} by 10% at $De_{e,2} = 1$, therefore

$$f(De_{e,2}) = 1 + 0.1\, De_{e,2} = 1 + 0.1 \frac{\Theta_{f,e} D_w}{2} \quad . \tag{26}$$

Then, the effective fluid characteristic time $\Theta_{f,e}$ can be determined from the slope of experimentally obtained dependence (*26*).

The values of the time $\Theta_{f,e}$ evaluated for individual test fluids from our experimental dependencies of ratio $\Delta p_{ve}/\Delta p_{pv} \equiv f(De_{e,2})$ upon variable D_w are listed in Table II. In order to compare the experimental pressure drop data with the predicted ones, the relative deviations between experimental data and those calculated according to the right-hand side term of Eq. (*13*) together with Eq. (*26*), making use of the values of $\Theta_{f,e}$ given in Table II, were determined. The mean relative deviations obtained for the individual systems investigated ranged from 4% to 22%. The agreement between experimental elastic pressure drop increase and that calculated according to Eq. (*26*) achieved for fixed bed measurements with solutions of Natrosol, Kerafloc, Separan, and starch sugar syrup with Separan is also evident from Fig. 4 in which the calculated data are displayed by full lines. Good agreement between experimental and calculated data documents the applicability of Eqs (*13*) or (*14*) along with Eq. (*26*) for the pressure drop prediction in the creeping flow of viscoelastic polymer solutions through fixed beds of spherical and non-spherical particles if the characteristic flow process time $\Theta_{p,2}$ and effective fluid time $\Theta_{f,e}$ are used.

However, it can be hardly expected that Eq. (*26*) will be valid out of the investigated ranges of experimental conditions ($1 \,.\, 10^{-8} < Re_n < 2$; $0 < De_{e,2} < 242$). It was found[23] that the dependence $\Delta p_{ve}/\Delta p_{pv}$ versus Deborah number based on a constant value of characteristic fluid time goes through a maximum at higher values of Deborah as well as Reynolds numbers. But such values were not reached in our experiments and Eq. (*26*) describes only the increasing part of dependence $\Delta p_{ve}/\Delta p_{pv}$ versus $De_{e,2}$.

The values of the effective fluid time $\Theta_{f,e}$ (Table II) were compared with the values of the characteristic fluid time $\Theta_{f,1}(\dot{\gamma})$, calculated according to Eq. (*18*) for the range of $\dot{\gamma}$ reached in our fixed bed flow experiments, and with the values of the time $\Theta_{f,1}$ determined for the value of shear rate corresponding to the condition $De_{e,2} = 1$. The resulting intervals of $\Theta_{f,1}(\dot{\gamma})$ and the "onset" values $\Theta_{f,1}$ are also summarized in Table II. It can be seen that the values of $\Theta_{f,1}$ fall into the intervals of $\Theta_{f,1}(\dot{\gamma})$ and the values of $\Theta_{f,e}$ and $\Theta_{f,1}$ are comparable for all fluids tested, except of Polyox solutions. The relation observed between values of the effective fluid time $\Theta_{f,e}$ and the characteristic time $\Theta_{f,1}$, derived from the corotating Maxwell model, shows that the time $\Theta_{f,e}$ can be regarded as a realistic measure of elastic behaviour of viscoelastic fluids flowing through a fixed bed of particles or through a medium with channels of similar geometry (converging-diverging channels). In this connection, a possibility arises to use the simple fixed bed rheometer for on-line rheometry purposes[28].

TABLE II
Characteristic of polymer solutions

No.	Composition[a]	ρ_f kg m^{-3}	Parameters of power law models					$\Theta_{f,e}$ s	$\Theta_{f,l}$(onset) s	range of $\Theta_{fl}(\dot{\gamma})$ s
			n	K Pa s^n	m	K_N Pa s^m	range of $\dot{\gamma}$ s^{-1}			
1	0.35% Separan AP-45	1 001	0.423	1.081	0.681	5.126	8.2–733.7	0.281 ± 0.070	0.373	0.017–0.498
2	0.5% Separan AP-45	1 003	0.389	1.983	0.667	12.60	7.3–1 326	0.141 ± 0.009	0.132	0.018–0.756
3	0.4% Kerafloc A4008	1 001	0.419	1.764	–	–	2.01–976.3	0.048 ± 0.011	–	–
4	1.4% Natrosol 250 MR	1 004	0.516	4.486	0.396	6.338	16.2–1 328	0	–	0.0002–0.0312
5	1.4% Polyox WSR 301	1 002	0.510	2.182	1.114	0.643	2.7–242.6	0.281 ± 0.070	0.078	0.017–0.100
6	1.5% Polyox WSR 301	1 002	0.430	4.759	0.672	17.22	16.2–437	0.122 ± 0.058	0.037	0.018–0.219
7	Boger fluid I (0.019% Separan AP-45, 57.65% Starch sugar syrup)	1 290	0.975	0.154	0.740	2.601	5.4–364.5	0.258 ± 0.007	0.720	0.006–1.052
8	Boger fluid II (0.024% Separan AP-45, 57.16% Starch sugar syrup)	1 287	0.980	0.141	0.826	1.286	5.4–364.5	0.112 ± 0.005	0.197	0.005–0.651

[a] Concentration in wt.%.

CONCLUSIONS

The pressure drop in the creeping flow of viscoelastic polymer solutions through fixed beds of spherical and non-spherical particles has been measured and a pressure drop excess due to fluid elasticity has been observed. Analysing the experimental data, a simple form of the corrective Deborah number function has been suggested enabling relations valid for the fixed bed flow of purely viscous non-Newtonian fluids to be modified for the flow of viscoelastic fluids. Procedures for estimating the necessary fluid and flow process times have also been proposed.

SYMBOLS

a_p	specific surface of particles, m^{-1}
a_w	specific surface of apparatus walls, m^{-1}
A, B	parameters in Eq. (*15*), dimensionless
d_p	particle diameter, m
D_h	hydraulic diameter of apparatus, m
D_w	kinematical variable, s^{-1}
De	Deborah number defined by Eq. (*1*), dimensionless
$De_{e,2}$	Deborah number expressed by Eq. (*24*), dimensionless
$De_{0.2}$	Deborah number expressed by Eq. (*23*), dimensionless
$De_{1.1}$	Deborah number expressed by Eq. (*20*), dimensionless
$De_{1.2}$	Deborah number expressed by Eq. (*22*), dimensionless
f	bed drag coefficient, dimensionless
K	flow curve power law model parameter, dimensionless
K_N	power law parameter of first normal stress difference, Pa s^m
l_{ch}	characteristic linear dimension of the bed void space, m
L	length of the bed, m
L_{ch}	characteristic path of fluid particle, m
M	corrective coefficient for the wall effect, dimensionless
m	power law parameter of first normal stress difference, dimensionless
N_1	first normal stress difference, Pa
n	flow curve power law model parameter, dimensionless
Δp	pressure drop, Pa
Re_n	Reynolds number defined by Eq. (*12*), dimensionless
u	superficial fluid velocity, m s^{-1}
u_{ch}	characteristic fluid velocity, m s^{-1}
U_{ch}	characteristic velocity of fluid particle, m s^{-1}
$\dot{\gamma}$	shear rate, s^{-1}
$\dot{\gamma}(\tau)$	shear rate function, s^{-1}
ε	bed voidage, dimensionless
η	non-Newtonian viscosity, Pa s
η_0	zero shear rate viscosity, Pa s
λ	Carreau flow model parameter, s
Θ_f	fluid characteristic time, s
Θ_p	flow process characteristic time, s
$\Theta_{f,e}$	effective fluid characteristic time, s

$\Theta_{f,0}$	fluid characteristic time expressed by Eq. (*19*), s
$\Theta_{f,1}$	fluid characteristic time expressed by Eq. (*18*), s
$\Theta_{p,1}$	flow process characteristic time expressed by Eq. (*16*), s
$\Theta_{p,2}$	flow process characteristic time expressed by Eq. (*17*), s
ρ_f	fluid density, kg m^{-3}
ρ_p	particle density, kg m^{-3}
τ	shear stress, Pa
τ_w	dynamical variable, Pa
$\tau_{1/2}$	Ellis flow model parameter, Pa
φ	bed factor, dimensionless
Ψ_1	coefficient of the first normal stress difference, Pa s^2
$\Psi_{1,0}$	zero shear rate first normal stress difference coefficient, Pa s^2

Subscripts

ve	viscoelastic
pv	purely viscous
e	effective value
0	zero shear rate limit value

Superscript

*	limit value

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